# Impact Resistance of a Simply Supported Beam

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Structures on the moon are subject to many hazardous conditions that we do not face on earth such as moonquakes, extreme temperatures and frequent micro-meteorite impacts. This paper is focused on the latter of the three. To simulate these impacts on a lunar structure, a simply supported beam and a moving projectile will be assumed. While an impact of a micro projectile moving in the range of kilometers per second cannot be realistically simulated, a similar kinetic energy can be generated with a much heavier projectile moving at a slower speed.

In order to execute this experiment on the base layer of a multi-layer structure, a section of the top layers will be removed in order to allow for direct impact of the base layer as shown below:

|  |  |
| --- | --- |
|  | *Fig. 1*  *A geodesic dome having a pendulum dropped directly on a supporting structure.* |

In order to simplify this scenario, the problem can be reduced to a block of mass *m* moving with velocity *v* impacting a simply supported beam squarely at its midpoint.

|  |  |  |
| --- | --- | --- |
|  | *Fig. 2*  *Simply supported beam having a mass propelled at its midpoint.* |  |

The following equations focus on calculating the amount of stress induced in a simply supported hollow rod as the result of a centered kinetic impact. The hollow rod has length L and is supported on ends A and B. The impact will be delivered from a projectile with mass m and velocity v at point C on rod AB. Rod AB has an outer diameter of D1 and inner diameter of D2.

|  |  |  |
| --- | --- | --- |
|  | *Fig. 3*  *Free body diagram of centrally loaded beam.* |  |

The formula for kinetic energy is:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

The formula for strain energy is:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

Where y­max occurs at midpoint C on the simply supported beam. From the table of beam deflections and slopes *(Beer, 2015)* we find the maximum deflection for simply supported beams is:

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

Combining these equations yields:

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

Solving for P:

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

Measuring the distance from the central axis to the outside of the tube produces:

|  |  |
| --- | --- |
|  |  |

(6)

The equation for the moment of area of a ring is:

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

From here, a cut is made at point C on the rod and the moment is taken about point C.

|  |  |  |
| --- | --- | --- |
|  | *Fig. 4*  *Free body diagram of the simply supported beam cut at point C.* |  |

|  |  |  |
| --- | --- | --- |
|  |  | (9.1)  (9.2)  (9.3) |

Because the impact is at the center of the beam, substitutions can be made for the static load and the reaction at A to find

|  |  |  |
| --- | --- | --- |
|  |  | (10) |

These equations were then used to produce both an Excel spreadsheet and a MATLAB program to produce a set of tables to determine the amount of stress caused by different weights and velocities in different materials. The data assumes values: L = 2m, E = 70 GPa, D1 = 5.04cm, D2 = 3.81cm.

While using an impactor that generates velocity in a projectile using air pressure, magnetic fields or spring energy will produce consistent velocities, they also come at a higher fabrication cost and firing cost. Similar levels of consistency can be achieved with a pendulum impactor that uses gravity as its mode of velocity generation. To derive this formula, the value for energy is replaced with

|  |  |  |
| --- | --- | --- |
|  |  | (11) |

This changes the equation for equivalent static load to:

|  |  |  |
| --- | --- | --- |
|  |  | (12) |

This formula was then cross referenced with equations 15 and 16 into the calculator in order to generate the following chart:

Given the stress induced in the aluminum beam, the deflection in the simply supported beam as well as the axial force and stress can be found. This can be calculated using the formula for maximum deflection from a table of beam deflections *(Beer, 2015)*.

|  |  |  |
| --- | --- | --- |
|  |  | (13) |

The previously derived values of *F* and *I* can be plugged into equation 13. The value for *E* will be taken from Appendix B for 6061 aluminum *(Beer, 2015)*.

This formula will generate a maximum deflection in the simply supported beam, but without knowing the limits of the material, these deflections will not mean anything. Cross referencing equation 17 with equation 12 shows that both equations use static force in the first power in the numerator. This means that equation 12 can be solved for force instead of stress:

|  |  |  |
| --- | --- | --- |
|  |  | (14) |

This equation can be used to calculate how much the beam will deform when it reaches maximum tensile stress.

|  |  |  |
| --- | --- | --- |
|  |  | (15) |

In this example, the maximum tensile deflection is found to be 4.91cm.

Using these formulas, the following charts were generated showing beam deflection over projectile velocity and drop height:

# Axial Impact Resistance of a Circular Beam

Thestress induced in a simply supported beam form a projectile fired at its center does not fully encapsulate all of the stresses that a lunar structure will experience. Another important mode of failure is by axial force, stress and deformation. In order to test the axial rigidity of a cylindrical beam, it will be connected to the inner layer of a lunar structure and impacted as shown in the diagram below.

|  |  |
| --- | --- |
|  | *Fig. 5*  *A geodesic dome having a pendulum dropped directly on an attached cantilever beam* |

In order to simplify this scenario, the problem can be reduced to a block of mass *m* moving with velocity *v* impacting a cantilevered beam squarely at its end.

|  |  |  |
| --- | --- | --- |
|  | *Fig. 6*  *Cantilevered beam having a mass propelled at its end.* |  |

In the following calculations, an impact on face A resulting in a distributed static load P orthogonal to face A will be assumed.

|  |  |
| --- | --- |
|  | *Fig. 7*  *Free body diagram of axial loaded beam.* |

This scenario will assume a 6061-aluminum beam of the same 2 meter length, 2 inch outer diameter and 1.5 inch inner diameter. Impact force will be provided along the axis of the tube either by a kinetic impactor or by a dropped pendulum.

To find the equivalent static load on our tube, the strain energy induced in the rod must first be calculated using the formula:

|  |  |  |
| --- | --- | --- |
|  |  | (16) |
|  |  |  |

This equation is then combined with equation 1 for kinetic energy, and equation 2 for strain energy to derive:

|  |  |  |
| --- | --- | --- |
|  |  | (17) |

This formula can also be cross referenced with equation 13 for gravitational potential energy giving:

|  |  |  |
| --- | --- | --- |
|  |  | (18) |

Both of these equations are then plugged into the equation for stress caused by an axial load:

|  |  |  |
| --- | --- | --- |
|  |  | (19) |

Using these equations, the following charts were generated depicting stress induced from kinetic and potential impacts over different firing speeds and drop heights:

These charts show that in practice, it is not realistic to generate an impact strong enough to permanently deform our aluminum rod with an axial impact from a dropped projectile like a pendulum as it would require in excess of a 100kg mass raised to a 5-meter height. While extremely difficult and expensive to deform the beam with a kinetic impactor, it is theoretically possible given the constraints of our experiment.

In addition to stress, it is important to calculate the displacement the beam will experience from these impacts as they will cause the most stress on the joints between the beams. To find the displacement in the member from an axial impact, the following equation is used:

|  |  |  |
| --- | --- | --- |
|  |  | (20) |

In order to get a sense of the resulting displacement values, equations 23 and 24 are combined to find the amount of displacement required to reach the maximum tensile load.

|  |  |  |
| --- | --- | --- |
|  |  | (21) |

Both of these equations were then used to generate the following charts:

Analyzing these charts reflects the conclusion from the earlier charts for stress from axial loads. It will be nearly impossible to generate enough stress to permanently deform the aluminum rod with a potential energy impactor given the constraints of the experiment while it will be possible but very difficult with a kinetic impactor.

# Pendulum Drop Height Optimization

The maximum drop height of an impactor pendulum is limited by two main factors: the height of the lab ceiling and the amount of weight we can realistically raise. Although it would be theoretically possible to raise upwards of 200kg with a complex pulley system or hydraulic assistance, it would not be realistic. In addition, all of the equations show an inverse exponential relationship between the drop height of the pendulum and the equivalent static load, meaning energy will see diminishing returns with raising the drop height of the pendulum. The shape of this curve can be generated by setting the values for potential and kinetic energies from equations 1 and 11 equal to each other giving:

|  |  |  |
| --- | --- | --- |
|  |  | (22) |

To find the optimal drop height of our pendulum to maximize impact velocity with a constrained ceiling height, the following graph was generated.

Analyzing the curve of impact velocity over drop height shows that while a drop height above 5 meters will provide a minimal increase in velocity, a drop height of 1 meter would not produce enough energy to be effective. The optimal drop height of the pendulum impactor will likely be in the 3.5 meter range before diminishing returns are experienced.

# References

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| [1] | S. Timoshenko, G. H. MacCullough, *Elements of Strength of Materials (3rd Edition)*, Van Nostrand Company, Inc., New York, NY, 1949. |
| [2] | F. Beer, E. Johnson, J. DeWolf, D. Mazurek, *Mechaincs of Materials Seventh Edition.* McGraw-Hill Education, New York, NY, 2015. |